Problem 1 (max 8 points)

Power stage of a voltage-fed inverter with dq-domain output-current control is as shown in Figure 1.

a) Develop the inverter average model in dq-domain at open-loop, \( \frac{3}{2} \delta \frac{d}{d t} \langle i_d \rangle + \frac{3}{2} \delta \frac{d}{d t} \langle i_q \rangle \).

b) Draw the equivalent linear circuit in dq-domain (DC input port and two AC output ports).

c) How do you have to modify the duty ratio \( d \) and \( q \)-components (by adding \( x_d \) and \( x_q \) ) to make the inverter output currents independent of the grid voltage \( d \) and \( q \)-components? I.e., how to implement grid voltage feedforward?

d) How do you realize decoupling of the current \( d \) and \( q \)-components? Use the average model to obtain necessary control laws / scaling coefficients.

![Figure 1: Voltage-fed inverter with output-current control.](image)

Problem 2 (max 4 points)

Instantaneous apparent power can be defined as the product of voltage space-vector and the complex-conjugate of current space-vector in the stationary reference frame as in (1).

a) Define real and imaginary power in the synchronous reference frame, i.e., in the dq-domain. The space vector is assumed to rotate at the fundamental grid frequency \( \omega_s \).

b) Explain (based on the previous result), how the real and imaginary power produced by three-phase inverter can be controlled independently.

\[
S = v^{\omega} \cdot (i^{\omega})^* \tag{1}
\]

Problem 3 (max 6 points)

Control block diagram of a phase-locked-loop is as shown in Figure 2. The feedforward term \( \omega_{ff} \) is a constant which improves start-up. The Park’s transformation can be linearized as \( \dot{v}_q = \dot{v}_q - V_a \dot{\theta} \) where \( \dot{v}_q \) denotes the ideal grid voltage \( q \)-component.

![Figure 2: Phase-locked-loop.](image)
a) Draw the linearized control block diagram and define control loop gain of the PLL.

b) Solve transfer function from the reference input \( \dot{v}_e \) to the controlled variable \( \dot{v}_u \) in dq-domain.

c) The transfer function from reference to the controlled variable can be written as a second-order system as in (2). Find out the damping ratio \( \zeta \) and natural frequency \( \omega_n \) in terms of controller parameters. You can assume that the controller transfer function is as given in (3).

\[
G = \frac{2\xi\omega_n s + \omega_n^2}{(s^2 + 2\xi\omega_n s + \omega_n^2)}
\]  

(2)

\[
G_c = \frac{(-1) \cdot K(s/\omega_n + 1)}{s}
\]  

(3)

**Problem 4** (max 6 points)

Give short answers to following questions.

1. What is the main benefit of implementing current control in dq-domain?
2. How can one implement current control of three-phase inverter outside dq-domain?
3. Why do you need to solve steady-state operating point?
4. Can you stabilize converter which has a RHP-pole in its control dynamics? How?
5. Give short definition of a cascaded control scheme?
6. How should the PLL bandwidth be selected when grid voltages are unbalanced?

**Problem 5** (max 6 points)

Figure 3 shows current control loop gain with a simple unity-gain integrator as the controller (4). Comment on the stability and expected performance of the current control. Propose a controller transfer function to increase stability margins and explain how to select controller parameters (poles/zeroes/gains). The control loop gain should have a crossover frequency around few kilohertz. You don’t need to give specific values for controller parameters. You may sketch bode diagrams to justify the tuning process.

\[
L_{out} = G_{col-o} \cdot \frac{1}{s}
\]  

(4)

![Bode Diagram](image)

**Figure 3:** Current control loop gain.