MATH.APP.240 Fourier methods

Exam 12.12.2022   Examiner: Petteri Laakkonen

NB No materials or calculators are allowed. Remember to justify your solutions. If you get stuck on some detail, but know what you should do in general, write that down. A collection of formulas is given on the last page.

1. Consider the function $f(t) = \mathrm{rect}_1(t)(1 - |t|)$, if $t \in (-2, 2)$, and $f(t+4) = f(t)$ for all $t \notin \mathbb{Z}$.

a) Sketch the graph of the function on the interval $-6 \leq t \leq 6$.

b) Calculate the trigonometric Fourier series of $f(t)$.

c) Does the Gibbs phenomenon occur? If it does, at which points and what is the approximate maximum over/undershoot near these points?

2. The samples $\{g_0, \ldots, g_7\}$ of a 4-periodic function $f(t)$ were measured at the time instants $t = k/2$ where $k = 0, 1, \ldots, 7$. Then the following five elements of the discrete Fourier transform were calculated:

$$G_0 = 2,\ G_1 = 1 + j,\ G_2 = -j,\ G_3 = 1 + 2j,\ G_4 = -1.$$

a) Deduce the elements G_5, G_6, and G_7 of the discrete Fourier transform.

b) Calculate $f(2)$.

c) Approximate $f(t)$ using the Fourier series and the given information.

3. **a)** Find the Fourier transform of the functions

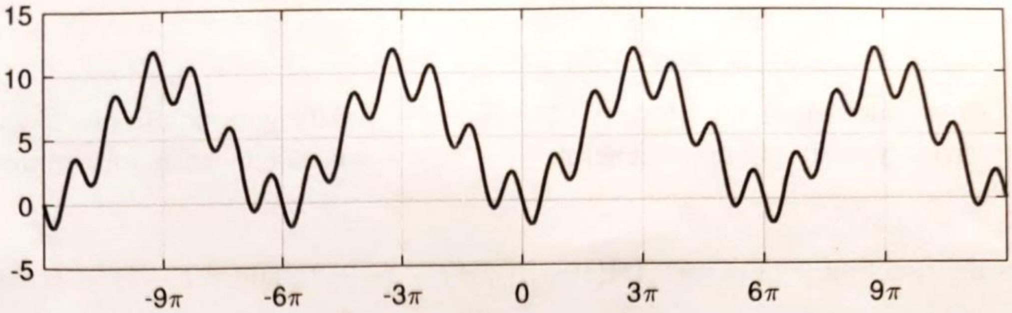
$$f(t) = \mathrm{sinc}(t-2) \quad \text{and} \quad g(t) = \frac{3(t-2)\cos(t-2) - 3\sin(t-2)}{t^2 - 4t + 4}.$$

Hint. Differentiate $f(t)$.

b) Prove the property $\mathscr{F}\{f(at)\}(\omega) = \frac{1}{|a|}F\left(\frac{\omega}{a}\right)$ when $a < 0$.

Problem 4 next page

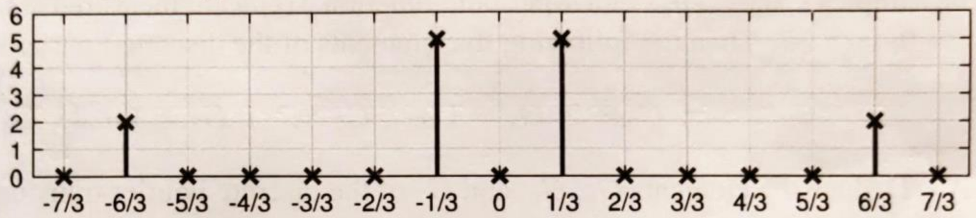
4. The graph of the periodic function $f(t)$ is given below.



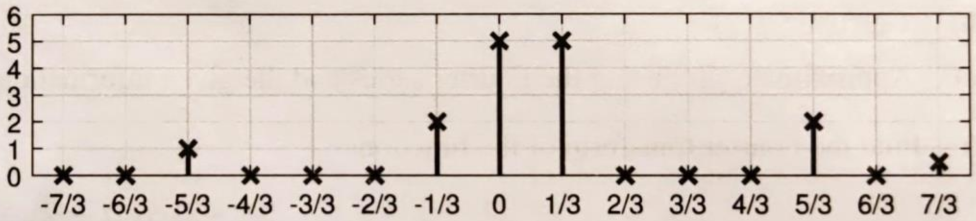
a) Which one of the figures below is the amplitude spectrum of $f(t)$? In the figures, the numbers of the horizontal axes denote the circular frequencies and the vertical axis denotes the value $|c_n|$. Justify your answer carefully.

b) Calculate the average energy of $f(t)$ using the amplitude spectrum you chose in **a**.

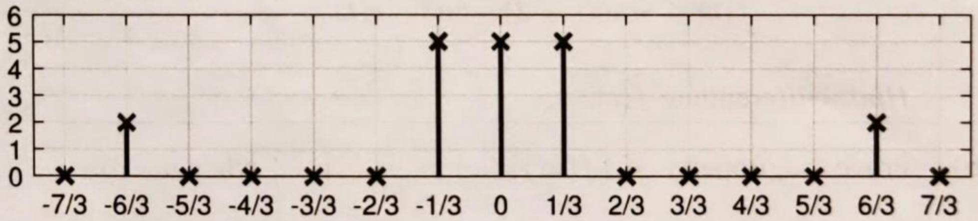
Spectrum 1:



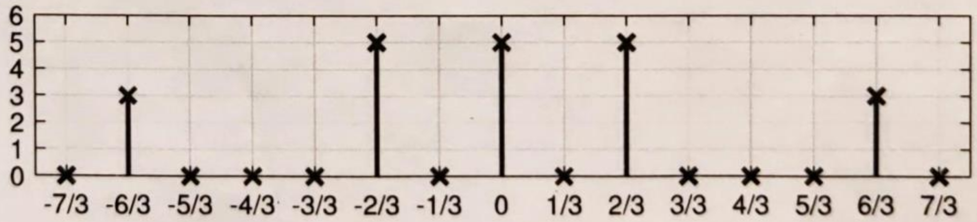
Spectrum 2:



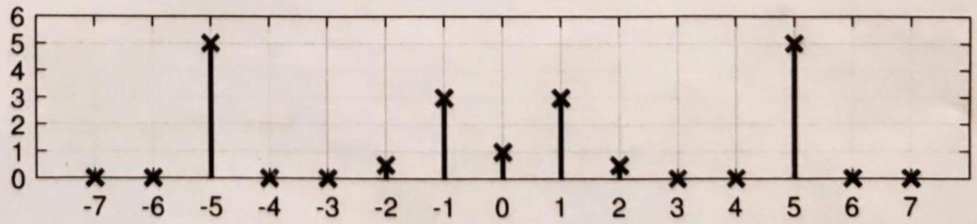
Spectrum 3:



Spectrum 4:



Spectrum 5:



Spectrum 6: